Fermi Gamma-Ray Bursts

Statistical Analysis and Coverage Estimation with GIT

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Supervised Learning Project

Abstract

Gamma-Ray Bursts are fast decaying transients that are fairly difficult to localize. SWIFT Observatory is one of the few telescopes that provide arcminute and sub-arcminute localizations, making follow-ups much easier. The Fermi Gamma-Ray space telescope provides localizations of the typical order of hundreds of square degrees. These large localizations make the follow-up of an optical afterglow difficult, requiring large field-of-view telescopes. This causes the majority of afterglows to go undetected, limiting our understanding of some of the brightest explosions in the universe. In this project, we will look at the capabilities of a meter-class telescope, such as the GROWTH-India Telescope (GIT) in following up on Fermi gamma-ray burst localizations. We'll also look at the follow-up results of an actual Fermi gamma-ray burst - GRB 231018A

Contents

Preliminaries

1.1 GROWTH-India Telescope

The GROWTH-India Telescope, part of the GROWTH (Global Relay of Observatories Watching Transients Happen) network is based in the Indian Astronomical Observatory (IAO) site at Hanle in Ladakh. Located at an altitude of 4500 meters, in an area officially declared as a dark-sky preserve, thereby providing excellent conditions to observe transient sources in the sky.

Some of the telescope's specifications:

- Telescope Planewave Instruments CDK700 with a 70cm aperture and a focal ratio of $f/6.5$ mounted on an Altitude-Azimuthal mount.
- Camera Andor iKon-XL is a back-illuminated cooled CCD with a resolution of 4108x4096 pixels at a resolution of 0.676 arcseconds per pixel, using the SDSS ugriz prime filters.

Figure 1.1: The GROWTH-India Telescope (Source: [https://sites.google.com/view/growthindia/gallery\)](https://sites.google.com/view/growthindia/gallery)

The operation of GIT is fully robotic, and it is the first of this kind in India. The telescope observations are jointly controlled by IIT-B and IIA. Daily targets and Targets of Opportunity (ToOs) for observation are uploaded remotely onto a server at the Hanle IAO site. The telescope then positions itself automatically to observe the designated part of the sky. The remote robotic operation is immensely helpful in allowing multiple groups of researchers to operate the telescope on the same night, while at the same time reducing the manpower needed on-site at Hanle.

1.2 Gamma-Ray Bursts

Gamma Ray Bursts or GRBs are intensely energetic electromagnetic events that have been observed in distant galaxies, believed to be primarily caused by supernovae. These events result in two narrow, collimated jets of gamma rays emanating along opposite directions. The initial burst of gamma rays can last from fractions of seconds to a few hours. These interact with the surrounding interstellar medium, energize them in turn, and result in this surrounding medium to emanate electro- magnetic radiation as well. This secondary emitted radiation, however, consists of longer wavelengths (X-ray, ultraviolet, optical, and infrared) and is called the 'afterglow' of the GRB. This afterglow lasts for significantly longer than the actual gamma-ray burst.

Figure 1.2: Types of emission from Gamma-Ray Bursts (Source: [https://svs.gsfc.nasa.gov/11407\)](https://svs.gsfc.nasa.gov/11407)

Gamma Rays cannot penetrate the Earth's atmosphere, hence the primary component of a GRB, if observable, must be observed using space telescopes. The Fermi Space Telescope, among others, is one of the primary searchers for GRB events in the sky. Since there is no prior knowledge of the location of the potential GRB events in the sky, it is built as a survey telescope, scanning the entire sky for possible transients. The network of gamma-ray space telescopes, on detecting an event, releases a notice of detection on the NASA GCN website. This enables ground-based telescopes such as GIT to follow up the original GRB by looking for its longer-wavelength counterparts from the afterglow.

1.3 Fermi Gamma-Ray Space Telescope

The Fermi Gamma-Ray Space Telescope [\(FGST\)](https://fermi.gsfc.nasa.gov/), formerly called the Gamma-ray Large Area Space Telescope (GLAST), launched on June 11th, 2008 is a space observatory being used to perform gamma-ray astronomy observations from low Earth orbit. Fermi carries two instruments: the Large Area Telescope [\(LAT\)](https://glast.sites.stanford.edu/) and the Gamma-ray Burst Monitor [\(GBM\)](https://gammaray.msfc.nasa.gov/gbm/). The LAT is Fermi's primary instrument, and the GBM is the complementary instrument. Both instruments provide alerts to GCN autonomously upon the detection of transients.

1.3.1 Large Area Telescope (LAT)

With its very large field of view, the LAT sees about one-fifth of the sky at any given moment. In sky-survey mode, which is the primary observing mode, the LAT will cover the entire sky every three hours. It operates with an unprecedented sensitivity to gamma rays in the 20 MeV to 300 GeV energy range with a field of view of 2.5 steradian and a localisation of less than 1° (statistical, 90%).

1.3.2 Gamma-ray Burst Monitor (GBM)

The Gamma-ray Burst Monitor (GBM) complements the LAT in its observations of transient sources and is sensitive to X-rays and gamma rays with energies between 8 keV and 40 MeV. It has a larger field of view of 8.8 steradians but also has a larger localization of $1 - 10^{\circ}$ (statistical + systematic).

1.3.3 General Coordinates Network (GCN)

The General Coordinates Network [\(GCN\)](https://gcn.nasa.gov/) is a public collaboration platform run by NASA for the astronomy research community to share alerts and rapid communications about high-energy transient phenomena.

GCN has two kinds of data products:

- [Notices](https://gcn.nasa.gov/notices) are automated, machine-to-machine, generally real-time, notifications of detections and localizations of astronomical transients detected by space- and ground-based observatories.
- [Circulars](https://gcn.nasa.gov/circulars) are human-readable, citable, rapid but generally not real-time, bulletins observations, quantitative near-term predictions, requests for follow-up observations, or future observing plans.

The Fermi team contributes to this General Coordinates Network by sending both Notices and Circulars regarding gamma-ray activity. The below table shows the various notices sent out using calculations done onboard or on the ground pipelines, along with the delay between the event occurring and the notice being sent.

Table 1.1: Types of notices sent by the Fermi team [\(https://gcn.gsfc.nasa.gov/fermi.html\)](https://gcn.gsfc.nasa.gov/fermi.html)

We also have the yearly trigger rates of both the Fermi instruments along with the type of trigger.

1.4 Problem Overview

The Fermi GBM and LAT detect over 250 gamma ray bursts yearly (refer Table [1.2\)](#page-6-2). Most of these bursts have a one-sigma statistical uncertainty of more than 1 degree in radius. Owing to this high uncertainty, optical follow-up for most observatory class telescopes is difficult, to say the least. Through a series of calculations, data collection, and statistical analysis, we aim to find out - "To what degree can a meter-class telescope, like GIT, be used to follow up Fermi GRB localizations?"

Instrument	Type	Rates
GBM	Short gamma-ray burst	$35 - 40$
	Long gamma-ray burst	200
	Soft gamma-ray repeater flares	$35 - 40$
	Terrestrial gamma-ray flashes	$80 - 90$
	Solar flares	$90 - 95$
L AT	Gamma-ray burst	$15 - 20$

Table 1.2: Yearly Trigger Rates [\(https://gcn.nasa.gov/missions/fermi\)](https://gcn.nasa.gov/missions/fermi)

1.4.1 Data Collection

The data products for all GRB triggers since operation are available at the [Fermi Trigger Information](https://gcn.gsfc.nasa.gov/fermi_grbs.html) page. For our analysis, we will take data products of roughly the past 5 years (1st January 2017 - 6th August 2023). The table of triggers was downloaded as a .csv file and localization files were retrieved from the [Fermi data archive.](https://heasarc.gsfc.nasa.gov/FTP/fermi/data/gbm/triggers/) Fermi also provides a list of confirmed GRBs on the [Fermi GBM Burst Catalog,](https://heasarc.gsfc.nasa.gov/w3browse/fermi/fermigbrst.html) which was also downloaded for the same time frame and analyzed.

1.4.2 Tiling

The GROWTH-India Telescope uses its own array of tiles to map the night sky and conduct follow-up observations of localisations greater than its field of view of 0.7 degrees. Using the localization maps, date, and time of each trigger, we were able to tile the localization of each trigger and produce statistics on the coverage and time taken to carry out the observations, assuming ideal conditions. An exposure time of 5 minutes per image was taken as the standard; observations began at the time of arrival of the trigger or when the first 90% confidence tile became visible.

Analysis

The analysis of GIT tiling on the localisations was done assuming the allocated time for each trigger was 1, 2, 3, or 4 hours as well as an ideal case of the entire nautical night.

2.1 Preliminary Analysis

Before getting into tiling each localization, the Fermi catalogs of real GRBs and the catalog of triggers provide us with some basic information about the distribution of triggers.

The total number of triggers for our time frame was 3766, out of which 1761 contained corresponding localization maps. The localization maps come in 2 formats - a regular fits file with pixel values containing the likelihood of detection, and a Healpix map containing a 2D probability map. There were 1409 healpix maps available for corresponding triggers as the healpix format was used by Fermi starting January 12, 2018.

The number of real GRBs confirmed by Fermi in their GBM Burst catalog was 1603, out of which 1499 had localization maps available. Fermi also classifies some of its triggers as "likely" Long and Short GRBs. The number of triggers reported as likely long and short GRBs were 948 and 241 respectively, out of which 907 and 214 of them were confirmed as actual GRBs.

2.2 Tiling Statistics

Tiling was done on each HEALPix localization map available for the triggers, along with calculations done for the 50% confidence area in each map.

Figure 2.1: Area vs Radius

We can see a clear trend in the actual area of localisations given in the HEALPix map when compared with the 1 sigma radius given by Fermi in their trigger notices.

Upon further inspection, we see 3 distinct paths followed by the area vs radius initially. This may be caused due to Fermi improving their localisations over the years.

Figure 2.2: Zoomed in Version of Figure [2.1](#page-7-3)

Upon plotting the localization area vs trigger date, we find the localization of triggers improved from about 65 square degrees to 30 square degrees in the last quarter of 2019.

Figure 2.3: Area vs Trigger Date

2.2.1 Full Night Observations

The ideal case of tiling assumes we have the entire night available to us for tiling, or, at least the entire night till after the first 90% confidence tile rises. We get the following plot for this case.

Figure 2.4: Coverage vs Area for a Full Night's Observation

Looking at GRBs with a probability coverage (of the total sky-map) of more than 0.2, we find 245 localisations satisfy this criterion. However, we must look at the time taken to cover these probabilities.

Figure 2.5: Probability Coverage vs Time Taken

We see that 20% probability is being covered only after spending 2 hours on each sky-map.

Using the graphs, we can come up with certain cuts observational for each trigger. The probability cutoff is used to select only those triggers which tiled to give a probability coverage greater than the cutoff. Total probability covered is simply the sum of the coverage of all the triggers above the cutoff, similarly, total observation time is the sum of times spent on each trigger above their respective cutoffs. Effective time per trigger is the total observation time divided by the total probability covered, the total probability essentially tells us the number of skymaps that would have been completely tiled and thus would have given a detection (assuming optically visible).

Probability Cutoff	Total Triggers	Total Probability Covered	Total Observation Time (hours)	Effective Time per Detection (hours)
0.05	607 (107.24)	120.9(21.35)	4288.2 (38.70%)	35.5(6.3)
0.10	433(76.50)	108.1(19.09)	3166.7 (28.58%)	29.3(5.2)
0.15	314 (55.48)	93.5(16.52)	2317.8 (20.92%)	24.8(4.4)
0.20	245 (43.29)	81.4 (14.39)	1864.9 (16.83%)	22.9(4.0)

Table 2.1: Time spent and probability covered for triggers with different coverage cutoffs. (Brackets contain yearly values and yearly percentage of GIT time for total observation time)

Probability Cutoff	Total Triggers	Total Probability Covered	Total Observation Time (hours)	Effective Time per Detection (hours)
0.05	360 (63.60)	79.7 (14.09)	2461.7 (22.21%)	30.9
0.10	279 (49.29)	73.6(13.01)	1993.1 (17.99%)	27.1
0.15	212 (37.46)	65.3(11.54)	1565.0 (14.12%)	24.0
0.20	174 (30.74)	58.8 (10.39)	1320.2 (11.91%)	22.4

Table 2.2: Time spent and probability covered for triggers with different coverage cutoffs - Long GRBs. (Brackets contain yearly values and yearly percentage of GIT time for total observation time)

Table 2.3: Time spent and probability covered for triggers with different coverage cutoffs - Short GRBs. (Brackets contain yearly values and yearly percentage of GIT time for total observation time)

2.2.2 One Hour Observations

Now, we will look at the same statistics if we had limited the observation time to 1 hour, naturally, it isn't feasible to give the entire night for each Fermi trigger. Further subsections will cover 2, 3, and 4-hour limits on observation time.

Figure 2.6: Coverage vs Area for 1-hour Observations

Looking at GRBs with a probability coverage (of the total sky-map) of more than 0.2, we find 0 localisations satisfy this criterion, however, there are 47 triggers with tiled localization greater than 0.1.

Figure 2.7: Probability Coverage vs Time Taken

We can see that the best coverage we get is close to 0.16, and nothing above 0.2.

Table 2.4: Time spent and probability covered for triggers with different coverage cutoffs. (Brackets contain yearly values and yearly percentage of GIT time for total observation time)

Probability Cutoff	Total Triggers	Total Probability Covered	Total Observation Time (hours)	Effective Time per Detection (hours)
0.05	148(26.15)	12.1(2.13)	147.3 (1.33%)	12.2
0.10	35(6.18)	4.4(0.77)	35.0 (0.32%)	8.0
0.15	2(0.35)	0.3(0.05)	$2.0(0.02\%)$	6.4
0.20	0(0.00)	0.0(0.00)	$0.0(0.00\%)$	nan

Table 2.5: Time spent and probability covered for triggers with different coverage cutoffs - Long GRBs. (Brackets contain yearly values and yearly percentage of GIT time for total observation time)

Probability Cutoff	Total Triggers	Total Probability Covered	Total Observation Time (hours)	Effective Time per Detection (hours)
0.05	4(0.71)	0.3(0.05)	4.0 (0.04%)	14.2
0.10	1(0.18)	0.1(0.02)	$1.0(0.01\%)$	9.3
0.15	0(0.00)	0.0(0.00)	$0.0(0.00\%)$	nan
0.20	0(0.00)	0.0(0.00)	$0.0(0.00\%)$	nan

Table 2.6: Time spent and probability covered for triggers with different coverage cutoffs - Short GRBs. (Brackets contain yearly values and yearly percentage of GIT time for total observation time)

2.2.3 Two Hour Observations

Now, we will look at the same statistics if we had limited the observation time to 2 hours.

Looking at GRBs with a probability coverage (of the total sky-map) of more than 0.2, we find 48 triggers satisfy this criterion, however, there are 192 triggers with tiled localisation greater than 0.1.

Figure 2.9: Probability Coverage vs Time Taken

We can see that the best coverage we get is close to 0.3.

Probability Cutoff	Total Triggers	Total Probability Covered	Total Observation Time (hours)	Effective Time per Detection (hours)
0.05	398 (70.32)	46.1 (8.15)	$813.4(7.34\%)$	17.6
0.10	192 (33.92)	31.1(5.50)	397.2 (3.58%)	12.8
0.15	98 (17.31)	19.8(3.49)	$203.1(1.83\%)$	10.3
0.20	48(8.48)	11.0(1.95)	99.8 (0.90%)	9.0

Table 2.7: Time spent and probability covered for triggers with different coverage cutoffs. (Brackets contain yearly values and yearly percentage of GIT time for total observation time)

Probability Cutoff	Total Triggers	Total Probability Covered	Total Observation Time (hours)	Effective Time per Detection (hours)
0.05	276 (48.76)	33.2(5.86)	561.4 (5.07%)	16.9
0.10	143 (25.27)	23.4(4.13)	295.5 (2.67%)	12.6
0.15	75(13.25)	15.1(2.67)	155.6 (1.40%)	10.3
0.20	36(6.36)	8.4(1.48)	$75.0(0.68\%)$	9.0

Table 2.8: Time spent and probability covered for triggers with different coverage cutoffs - Long GRBs. (Brackets contain yearly values and yearly percentage of GIT time for total observation time)

Table 2.9: Time spent and probability covered for triggers with different coverage cutoffs - Short GRBs. (Brackets contain yearly values and yearly percentage of GIT time for total observation time)

2.2.4 Three Hour Observations

Now, we will look at the same statistics if we had limited the observation time to 3 hours.

Coverage vs. Area (3 hours)

Figure 2.10: Coverage vs Area for 3-hour Observations

Looking at GRBs with a probability coverage (of the total sky-map) of more than 0.2, we find 95 triggers satisfy this criterion.

Figure 2.11: Probability Coverage vs Time Taken

We can see that the best coverage we get is close to 0.35.

Probability Cutoff	Total Triggers	Total Probability Covered	Total Observation Time (hours)	Effective Time per Detection (hours)
0.05	488 (86.22)	67.1 (11.86)	1413.2 (12.75%)	21.1
0.10	289 (51.06)	52.8(9.33)	852.2 (7.69%)	16.1
0.15	170 (30.04)	38.1(6.74)	501.4 (4.52%)	13.1
0.20	95 (16.78)	25.3(4.47)	$282.0(2.54\%)$	11.1

Table 2.10: Time spent and probability covered for triggers with different coverage cutoffs. (Brackets contain yearly values and yearly percentage of GIT time for total observation time)

Table 2.11: Time spent and probability covered for triggers with different coverage cutoffs - Long GRBs. (Brackets contain yearly values and yearly percentage of GIT time for total observation time)

Probability Cutoff	Total Triggers	Total Probability Covered	Total Observation Time (hours)	Effective Time per Detection (hours)
0.05	28(4.95)	2.6(0.47)	$82.8(0.75\%)$	31.3
0.10	8(1.41)	1.2(0.22)	24.0 (0.22%)	19.6
0.15	4(0.71)	0.8(0.14)	12.0 (0.11%)	15.6
0.20	1(0.18)	0.3(0.05)	$3.0(0.03\%)$	10.9

Table 2.12: Time spent and probability covered for triggers with different coverage cutoffs - Short GRBs. (Brackets contain yearly values and yearly percentage of GIT time for total observation time)

2.2.5 Four Hour Observations

Finally, we will look at the same statistics if we had limited the observation time to 4 hours.

Figure 2.12: Coverage vs Area for 4-hour Observations

Looking at triggers with a probability coverage (of the total sky-map) of more than 0.2, we find 144 triggers satisfy this criterion, and there are 59 triggers with coverage greater than 0.3.

Probability Covered vs Time (4 hours)

Figure 2.13: Probability Coverage vs Time Taken

Probability Cutoff	Total Triggers	Total Probability Covered	Total Observation Time (hours)	Effective Time per Detection (hours)
0.05	525 (92.76)	83.4 (14.73)	1979.9 (17.87%)	23.8
0.10	347(61.31)	70.3(12.42)	1340.2 (12.09%)	19.1
0.15	231(40.81)	56.0(9.90)	896.9 (8.09%)	16.0
0.20	144 (25.44)	40.8(7.21)	563.8 (5.09%)	13.8

Table 2.13: Time spent and probability covered for triggers with different coverage cutoffs. (Brackets contain yearly values and yearly percentage of GIT time for total observation time)

Table 2.14: Time spent and probability covered for triggers with different coverage cutoffs - Long GRBs. (Brackets contain yearly values and yearly percentage of GIT time for total observation time)

Probability Cutoff	Total Triggers	Total Probability Covered	Total Observation Time (hours)	Effective Time per Detection (hours)
0.05	33(5.83)	3.6 (0.64)	129.8 (1.17%)	35.7
0.10	14(2.47)	2.3(0.40)	56.0 (0.51%)	24.8
0.15	6(1.06)	1.3(0.22)	24.0 (0.22%)	19.0
0.20	3(0.53)	0.8(0.14)	12.0 (0.11%)	15.7

Table 2.15: Time spent and probability covered for triggers with different coverage cutoffs - Short GRBs. (Brackets contain yearly values and yearly percentage of GIT time for total observation time)

2.3 Comparison of Coverage

Table 2.16: Comparison of time spent and probability covered for triggers with different coverage cutoffs. (Brackets contain yearly values and yearly percentage of GIT time for total observation time)

Table 2.17: Comparison of time spent and probability covered for triggers with different coverage cutoffs - Long GRBs. (Brackets contain yearly values and yearly percentage of GIT time for total observation time)

Table 2.18: Comparison of time spent and probability covered for triggers with different coverage cutoffs - Short GRBs. (Brackets contain yearly values and yearly percentage of GIT time for total observation time)

Results and Conclusion

3.1 Follow-up Criteria

Using the final tables produced in Section [2.3,](#page-19-0) a follow-up criterion was decided with our partners at IIA.

- Proceed with follow-up if tiling predicts a coverage of more than 20% of the probability within 2 hours, for the case of Long GRBs.
- Proceed with follow-up if tiling predicts a coverage of more than 20% of the probability after spending a full night observing the trigger, for the case of Short GRBs.

3.2 GRB 231018A

At 12:30:10 UT on 18 Oct 2023, the Fermi Gamma-ray Burst Monitor (GBM) triggered and located GRB 231018A. Upon running the tiling algorithm, we got a predicted coverage of 22.2% within 2 hours of observations. We thus triggered GIT for follow-up of this GRB at 2023-10-18 17:28:08.912 UT, almost 5 hours later. Results of the coverage and upper limits obtained were published via NASA's GCN - [Circ. 34839.](https://gcn.nasa.gov/circulars/34839)

Figure 3.1: Fermi Localisation of GRB231018A, with the innermost contour representing GIT coverage

Other Work

Working in time domain astronomy has the upside (or downside) of any event happening at any time. Follow-up of such events as quickly as possible is the key to gaining valuable information about them. I have been involved in the observational aspect of the GROWTH-India Telescope, and its follow-up of GRBs, Minor planets, and other transients.

4.1 GIT230919aa: Discovery of an extragalactic Nova

- R. Kumar, V. Swain, et al., "GIT discovery of an optical transient in M31", AstroNote [2023-255](https://www.wis-tns.org/astronotes/astronote/2023-255)
- Judhajeet Basu, Ravi Kumar, et al., "Spectroscopic classification of AT 2023tkw (GIT230919aa) as a nova in Fe II phase", ATel [16311](https://www.astronomerstelegram.org/?read=16311)

4.2 GRB Follow-ups

- R. Kumar, et al., "GRB230816A: Possible host and GIT detection of optical counterpart", GCN [34460](https://gcn.nasa.gov/circulars/34460)
- R. Kumar, et al., "GRB 230812B: GIT optical follow-up", GCN [34420](https://gcn.nasa.gov/circulars/34420)
- R. Kumar, et al., " GRB 231018A: GROWTH-India Follow-Up of a Fermi Long GRB", GCN [34839](https://gcn.nasa.gov/circulars/34839)
- R. Kumar, et al., "GRB231017A: GROWTH-India upper limits on the optical afterglow", GCN [34833](https://gcn.nasa.gov/circulars/34833)
- R. Kumar, et al., "GRB 20230818A: GIT optical upper limit", GCN [34514](https://gcn.nasa.gov/circulars/34514)
- A. Salgundi, V. Swain, R. Kumar, et al., "AT2023sva / GRB230916B: GIT observations of the afterglow", GCN [34780](https://gcn.nasa.gov/circulars/34780)
- V. Swain, A Salgundi, R. Kumar, et al., "GRB 230827.256 : GIT optical follow-up of ZTF23abaanxz/AT2023qxj", GCN [34576](https://gcn.nasa.gov/circulars/34576)
- H. Kumar,..., R. Kumar, et al., "GRB 230812B: GIT Confirmation of SN rise", GCN [34500](https://gcn.nasa.gov/circulars/34500)

4.3 Minor Planets

- COMET P/2023 S1, MPEC [2023-S264](https://www.minorplanetcenter.net/mpec/K23/K23SQ4.html)
- COMET C/2023 R1 (PANSTARRS), MPEC [2023-R197](https://www.minorplanetcenter.net/mpec/K23/K23RJ7.html)
- 2023 RN3, [2023-R115](https://www.minorplanetcenter.net/mpec/K23/K23RB5.html)

Acknowledgements

It is an honor to be part of the GROWTH-India team and help out with the extremely important science being done. This project and the associated interactions have been an amazing learning experience.

I would like to thank Vishwajeet Swain and Dr. Harsh Kumar, who taught me the workings of GIT and involved me in the regular operations of the telescope. I extend my sincere gratitude, to Prof. Varun Bhalerao for allowing me to work on this project as a part of STAR Lab and overseeing my progress, as well as Prof. G. C. Anupama and Prof. Sudhanshu Barway from the Indian Institute of Astrophysics (IIA) for providing their valuable feedback throughout the project. Lastly, I would like to thank Tamojeet Roychowdhury for developing the tiling code which was used extensively in this project.